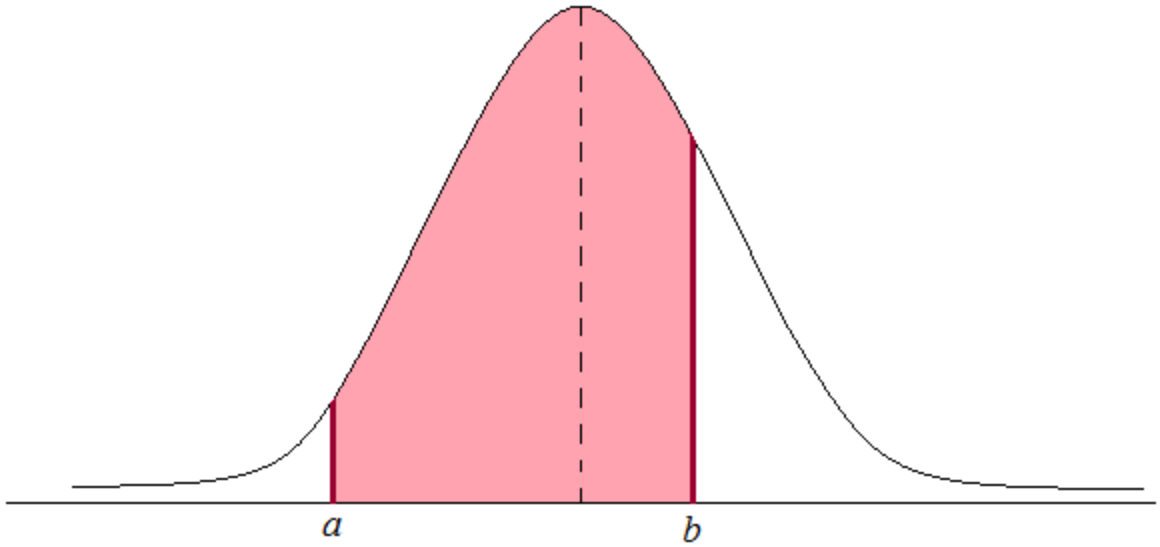
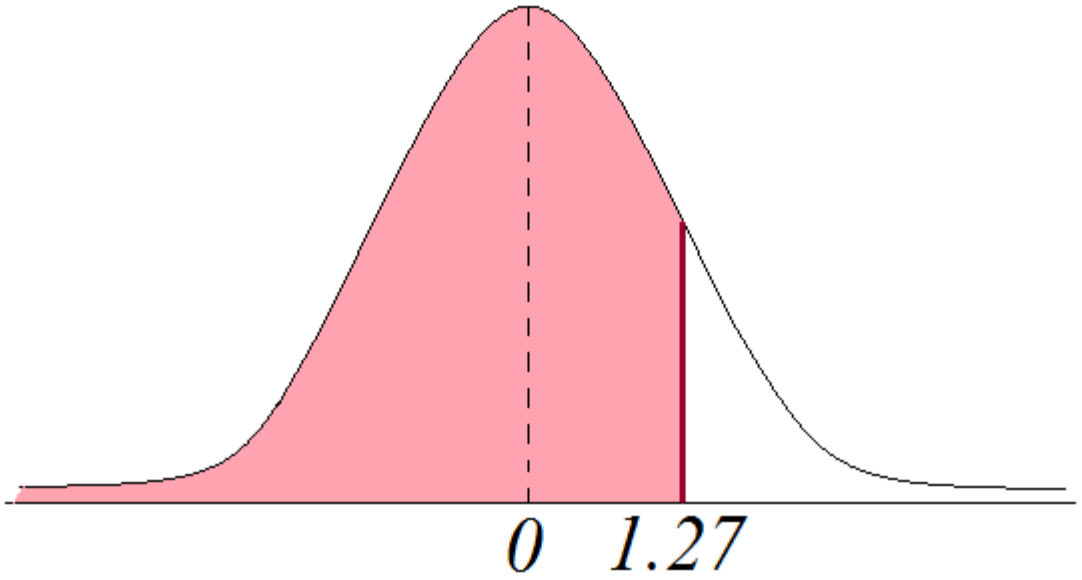
Sec. 7.1 and 7.2 (continued): Normal Distributions & Probabilities

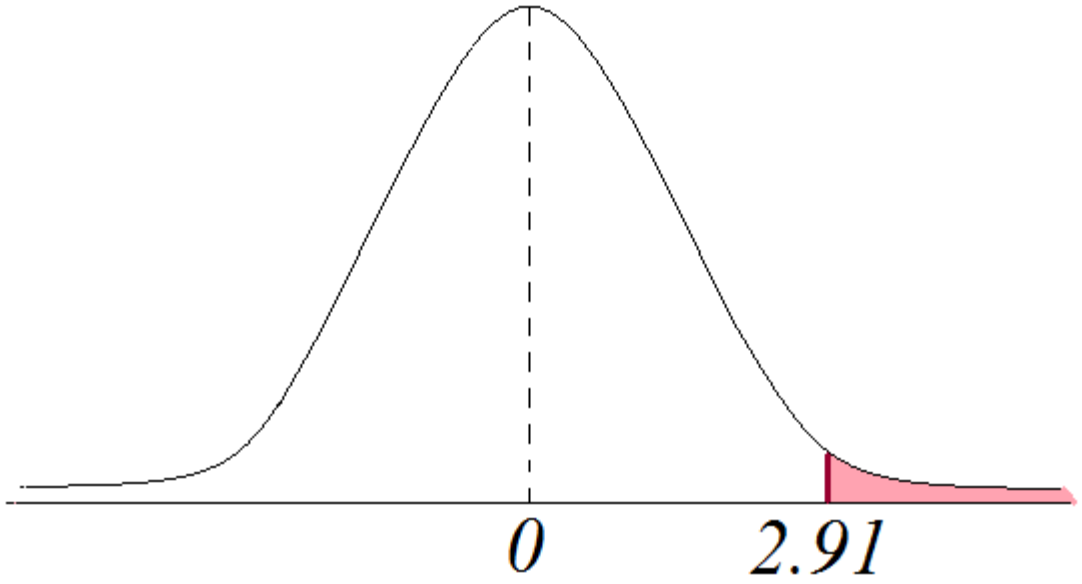


Notation Practice



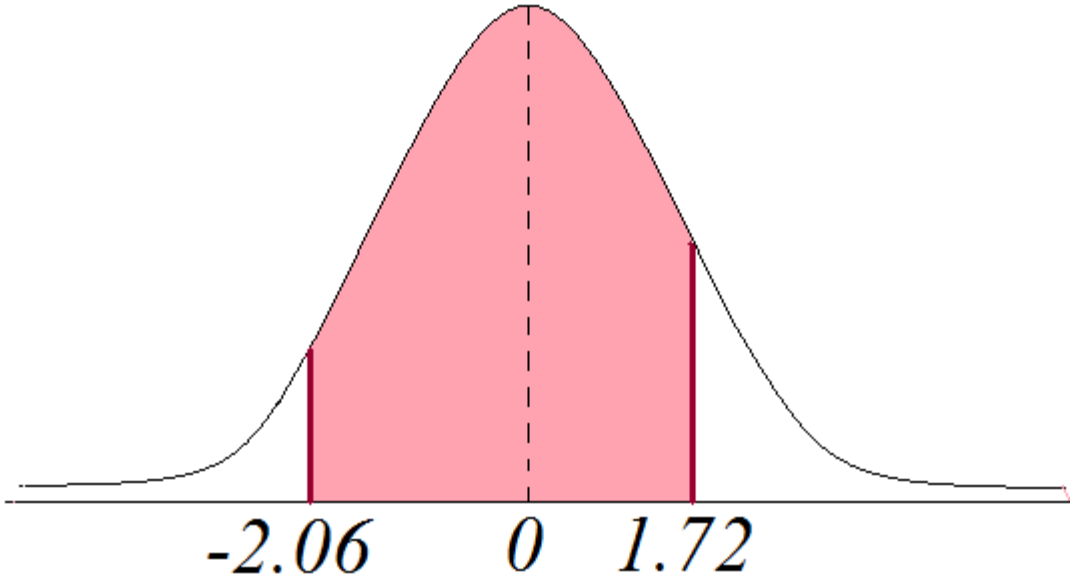
Notation: $P(Z < 1.27)$

Notation Practice



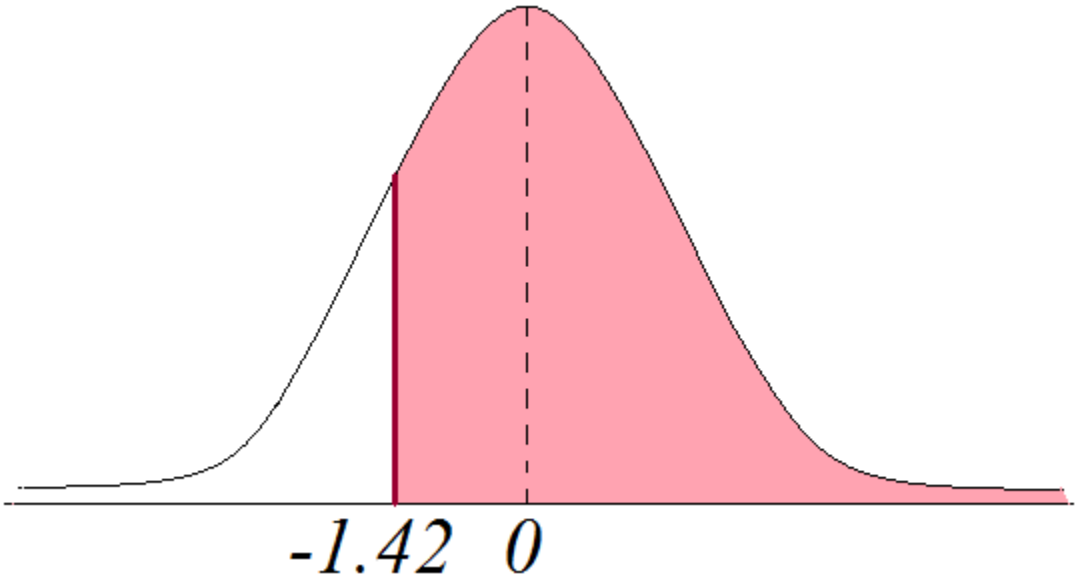
Notation: $P\,(\,Z > 2.91\,)$

Notation Practice



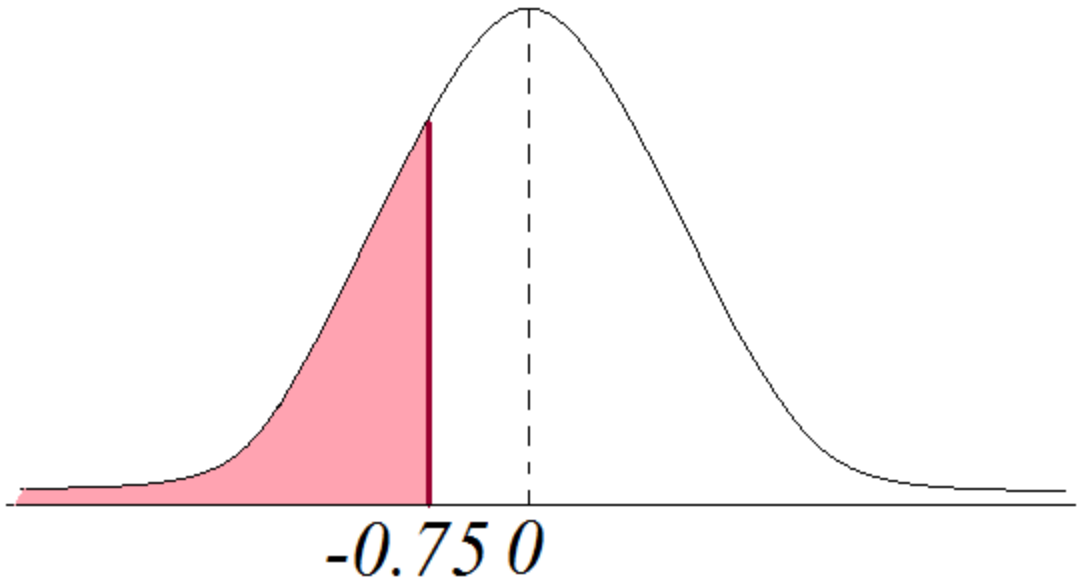
Notation: $P(-2.06 < Z < 1.72)$

Notation Practice



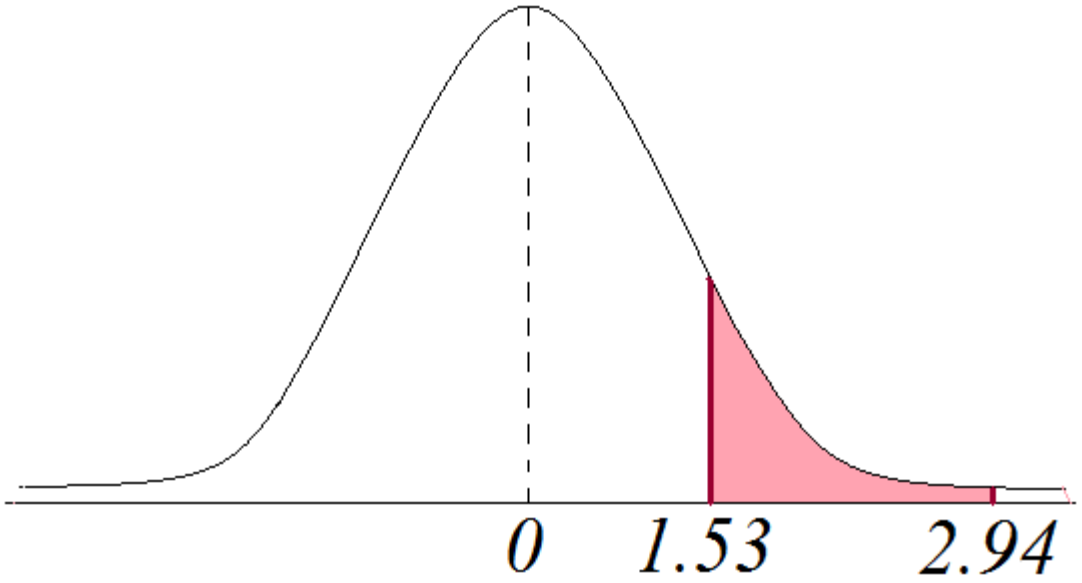
Notation: $P\,(\,Z > -1.42\,)$

Notation Practice



Notation: $P\,(\,Z < -0.75\,)$

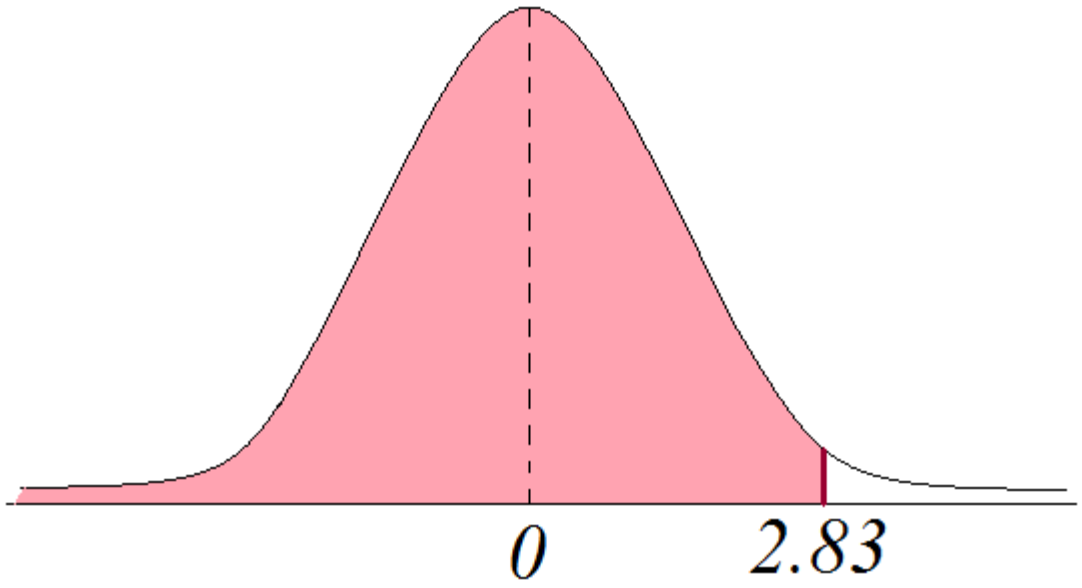
Notation Practice



Notation: $P\,(\,1.53 < Z < 2.94\,)$

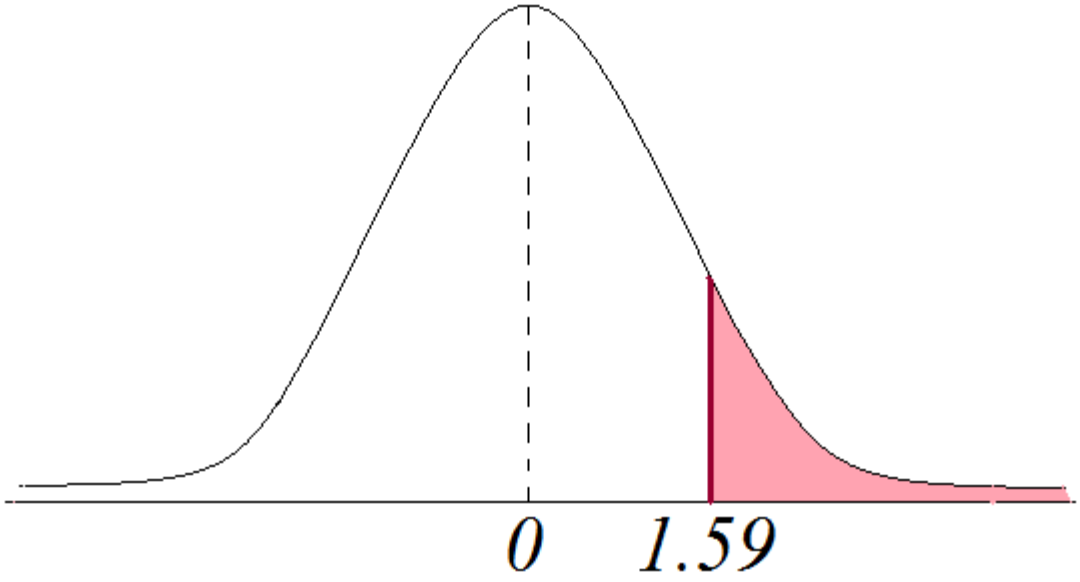
Picture Practice

Draw a picture for $P(Z < 2.83)$



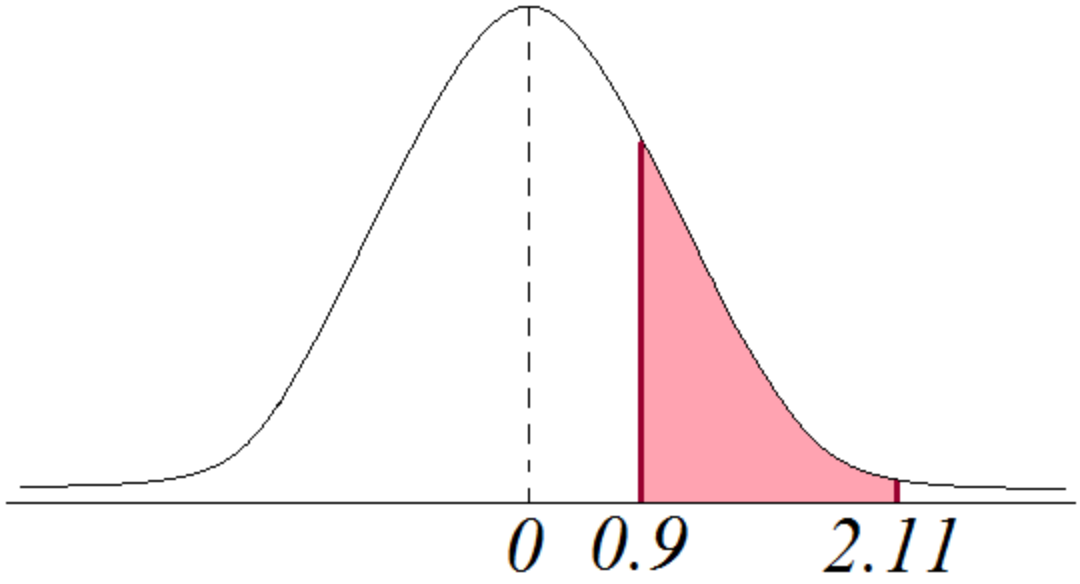
Picture Practice

Draw a picture for $P(Z > 1.59)$



Picture Practice

Draw a picture for $P(0.9 < Z < 2.11)$

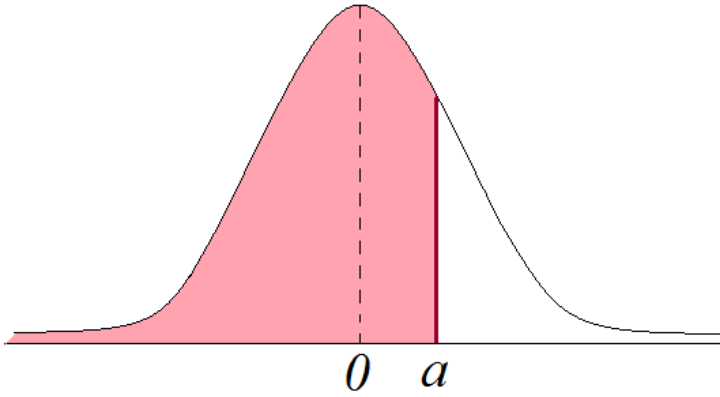


Finding Probabilities for the Standard Normal Random Variable Z

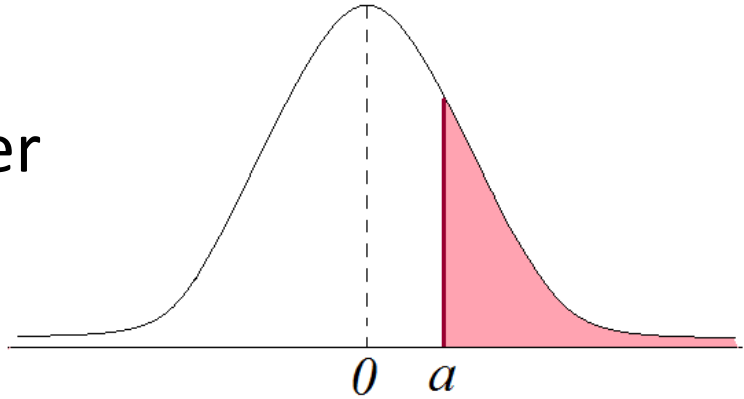
Finding Probabilities for Z

3 Types

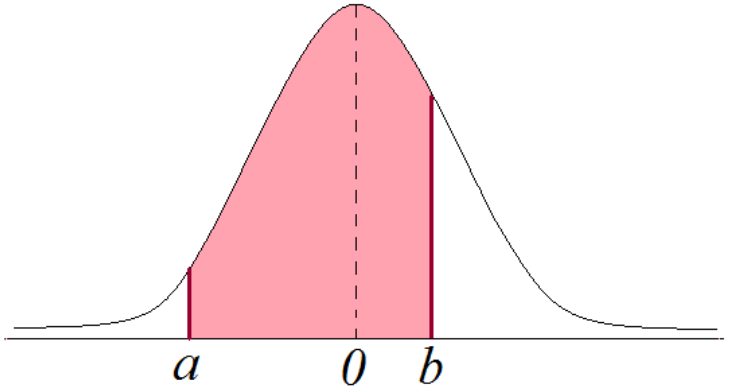
I) Probabilities to the left of a number



II) Probabilities to the right of a number

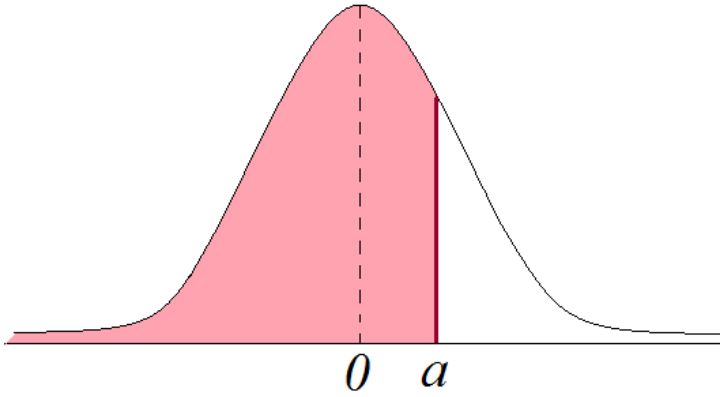


III) Probabilities between 2 numbers



Finding Probabilities for Z

I) To find probabilities to the left of a number…

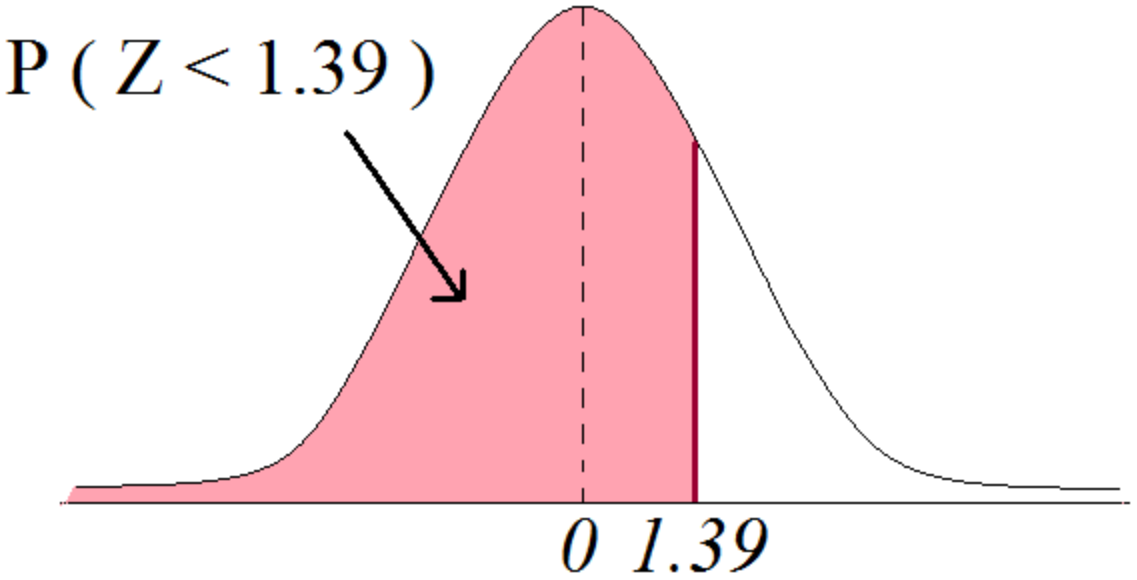


EASY!!! Just use the *Z*-table.

The table was made to answer this type of problem

Probabilities to the left of a number examples

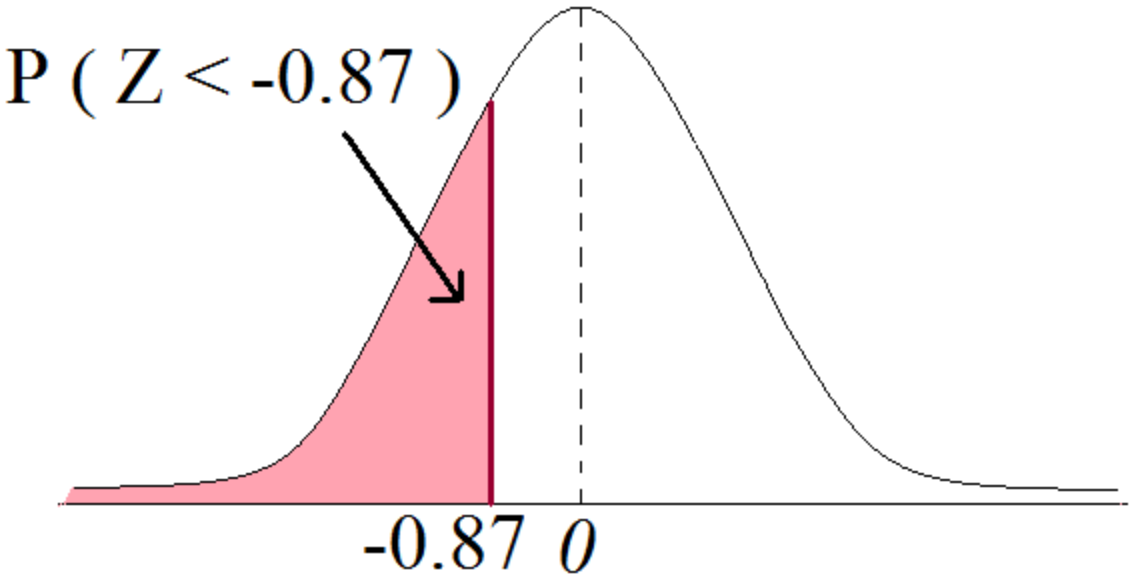
Ex 1: Find $P(Z < 1.39)$



Ans: $P(Z < 1.39) = 0.9177$

Probabilities to the left of a number examples

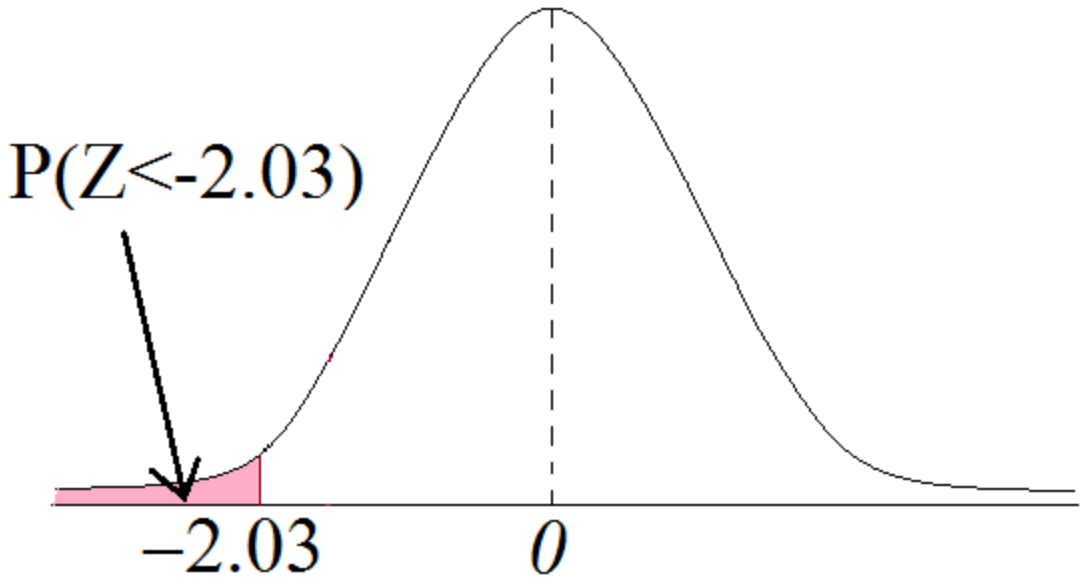
Ex 2: Find $P(Z < -0.87)$



Ans: $P(Z < -0.87) = 0.1922$

Probabilities to the left of a number examples

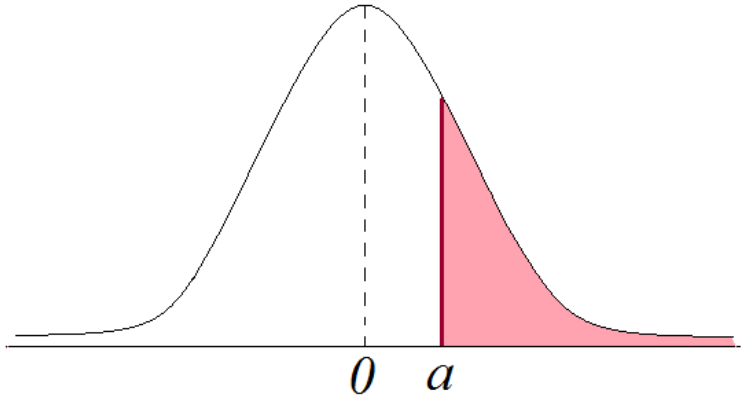
Ex 3: Find $P(Z < -2.03)$



Ans: $P(Z < -2.03) = 0.0212$

Finding Probabilities for Z

II) To find probabilities to the right of a number…

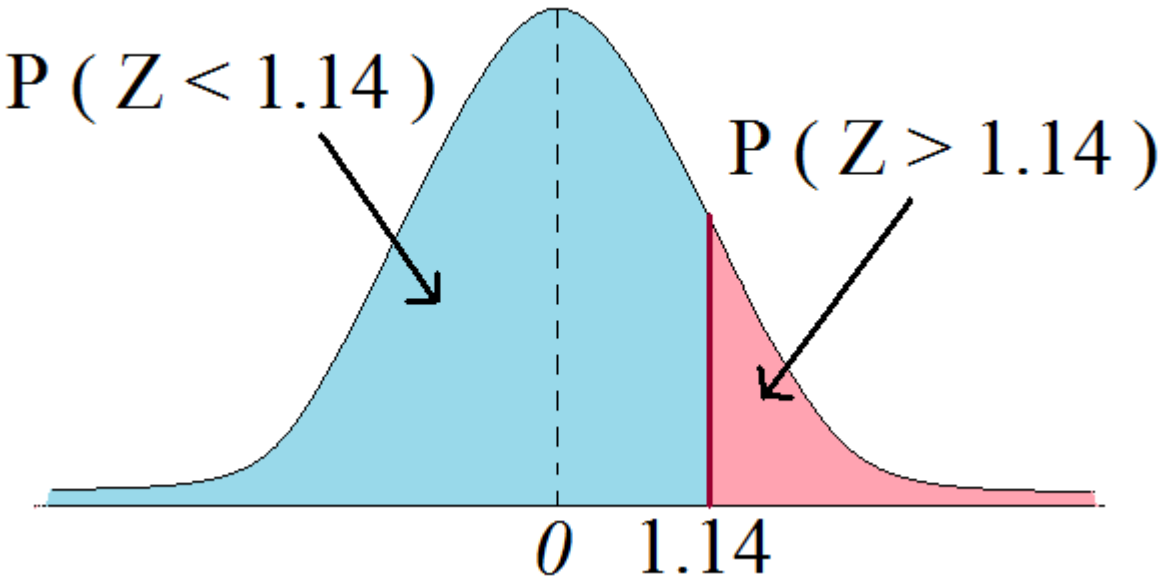


Not as easy, but still easy.

Turn it into a probability to the left with complements

Probabilities to the right of a number examples

Idea



$P(Z < 1.14) + P(Z > 1.14) = 1$

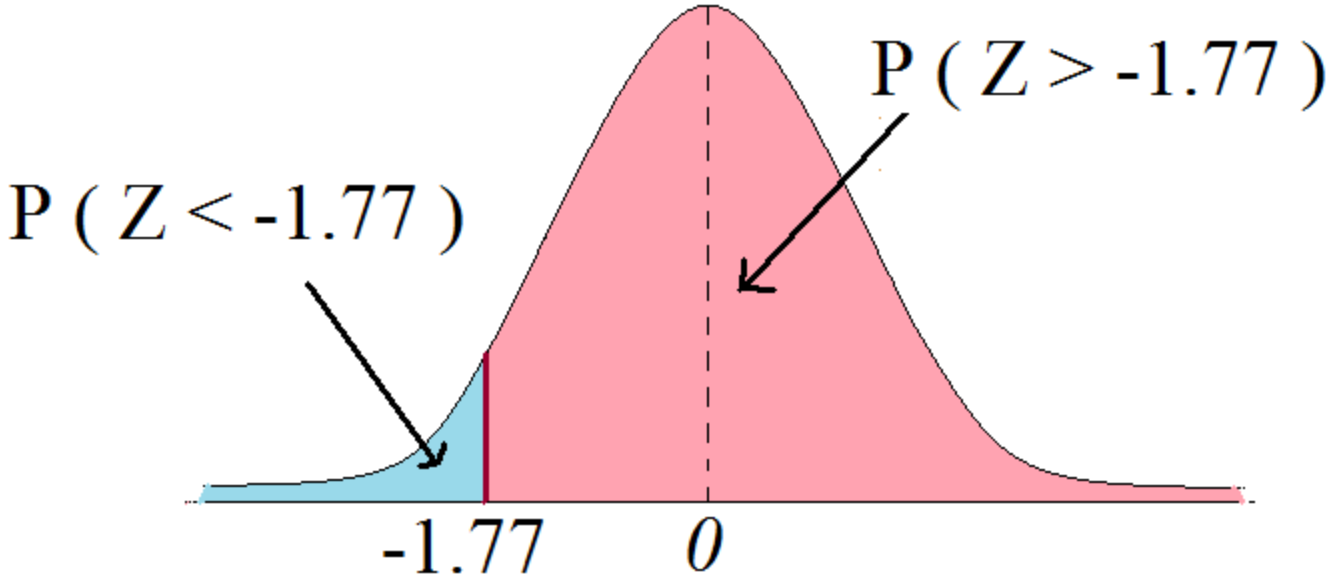
So… $P(Z > 1.14) = 1 - P(Z < 1.14)$

$= 1 - 0.8729$ (from table)

$= 0.1271$

Probabilities to the right of a number examples

Ex 4: Find $P(Z > -1.77)$

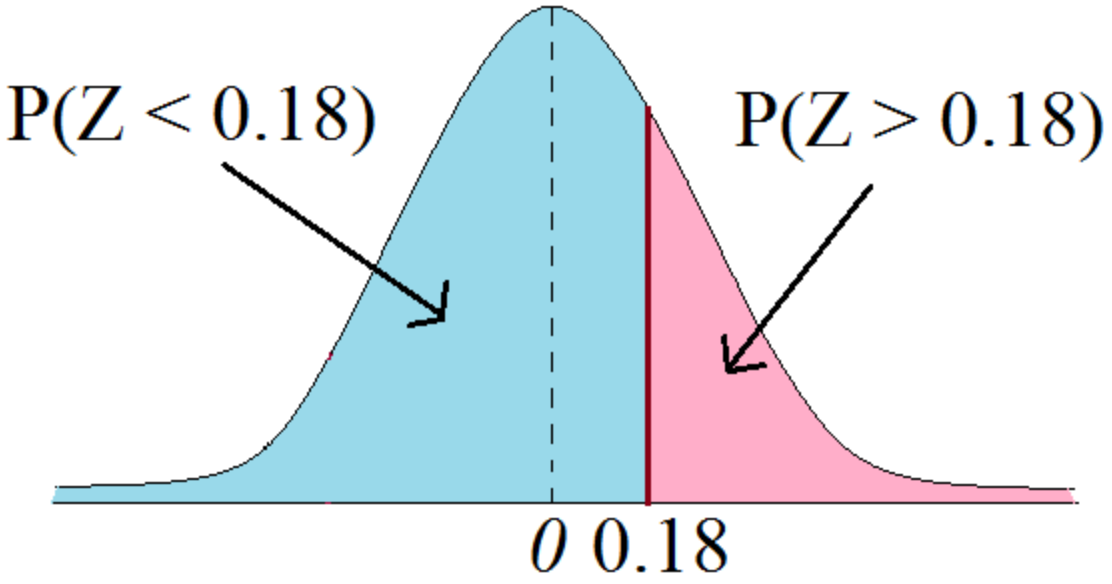


(table)

$P(Z > -1.77) = 1 - P(Z < -1.77) = 1 - 0.0384 = 0.9616$

Probabilities to the right of a number examples

Ex 5: Find $P(Z > 0.18)$

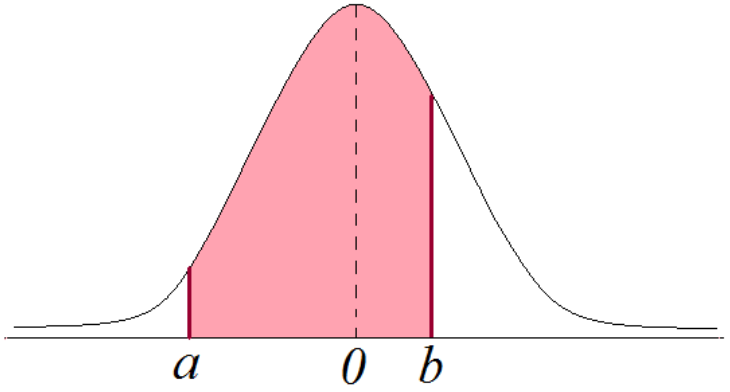


(table)

$P(Z > 0.18) = 1 - P(Z < 0.18) = 1 - 0.5714 = 0.4286$

Finding Probabilities for Z

III) To find probabilities between 2 numbers…

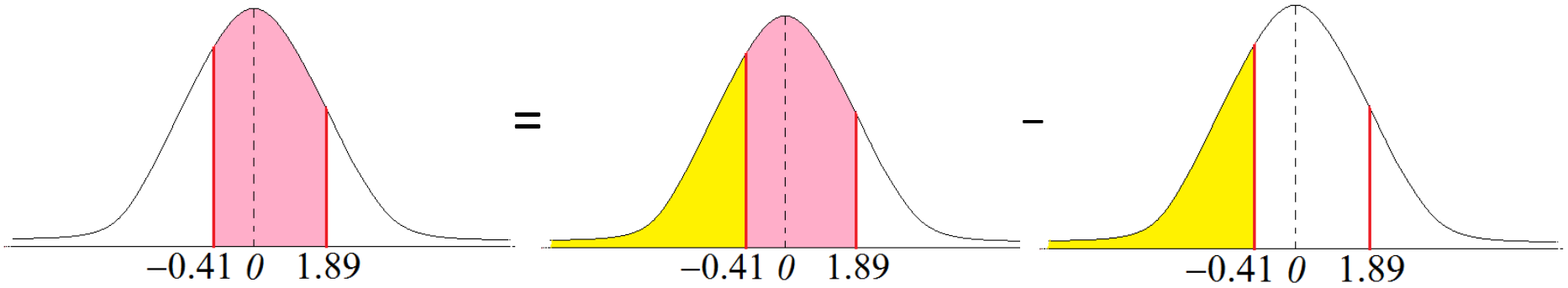


Most difficult one, but easy once you get it.

Turn it into a difference of 2 probabilities to the left

Finding Probabilities for Z

Idea



P (-0.41< Z < 1.89) = P (Z <1.89) – P (Z <-0.41)

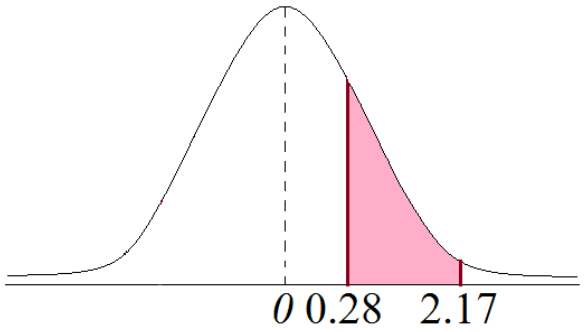
= 0.9706 – 0.3409

(from table) (from table)

= 0.6297

Finding Probabilities for Z

Ex 6: Find $P(0.28 < Z < 2.17)$



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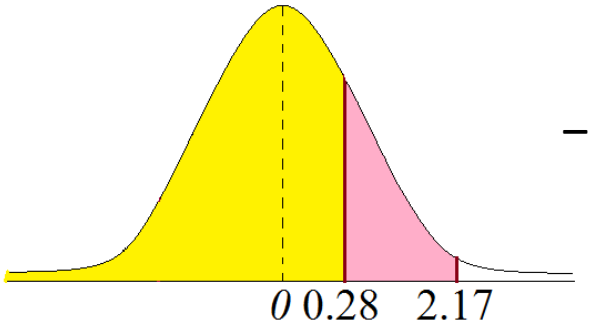

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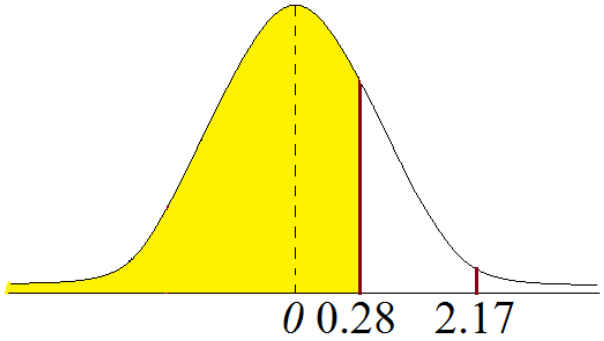


$P(0.28 < Z < 2.17) = P(Z < 2.17) - P(Z < 0.28)$

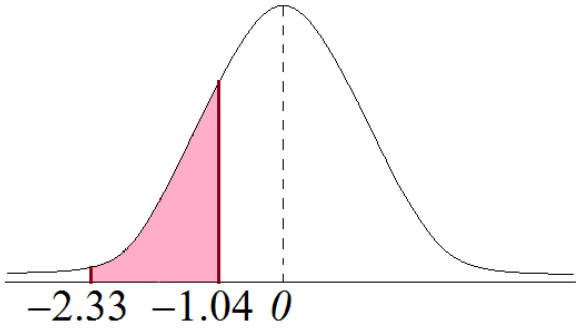
$= 0.9850 - 0.6103$

(from table) (from table)

$= 0.3747$

Finding Probabilities for Z

Ex 7: Find P(-2.33 < Z < -1.04)



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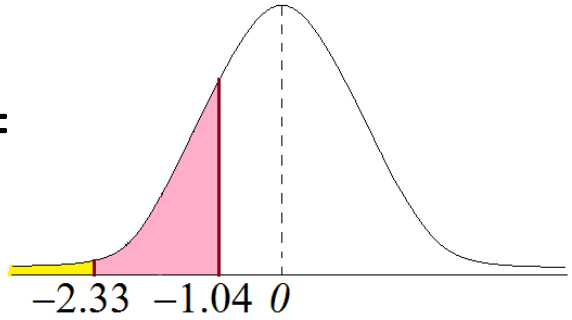

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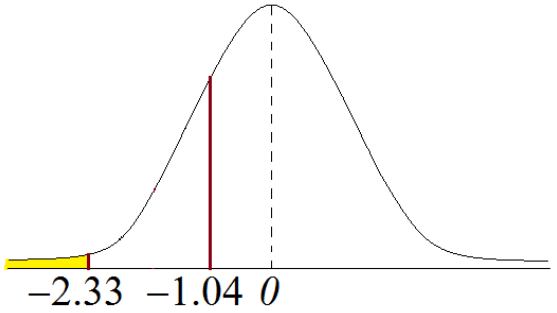


P (-2.33< Z <-1.04) = P (Z <-1.04) – P (Z <-2.33)

= 0.1492 – 0.0099

(from table) (from table)

= 0.1393

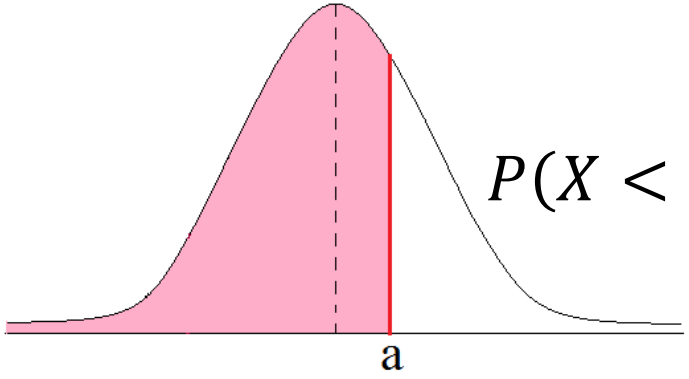
Finding Probabilities for X if X is Normal but not Standard Normal

Finding Probabilities for *X* if *X* is Normal but not Standard Normal

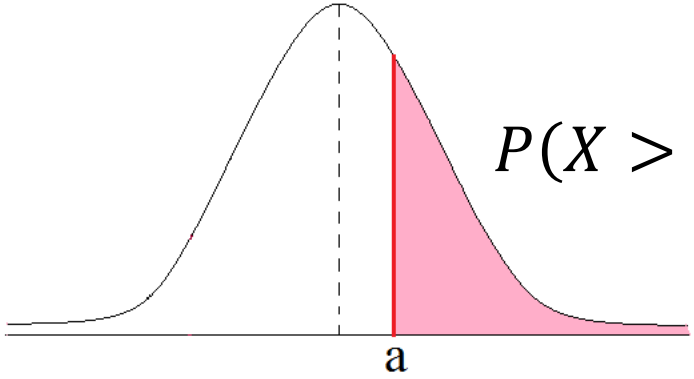
If the random variable *X* has a normal distribution with mean μ , standard deviation σ , but doesn't have the standard normal distribution, then turn the problem into a standard normal distribution problem using a *Z*-Transformation!

$$Z = \frac{X - \mu}{\sigma}$$

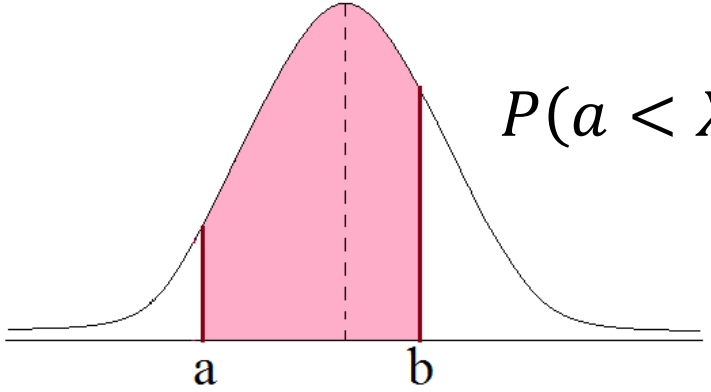
The *Z*-Transformation



$$P(X < a) = P\left(\frac{X-\mu}{\sigma} < \frac{a-\mu}{\sigma}\right) = P\left(Z < \frac{a-\mu}{\sigma}\right)$$



$$P(X > a) = P\left(\frac{X-\mu}{\sigma} > \frac{a-\mu}{\sigma}\right) = P\left(Z > \frac{a-\mu}{\sigma}\right)$$



$$P(a < X < b) = P\left(\frac{a-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{b-\mu}{\sigma}\right)$$

$$= P\left(\frac{a-\mu}{\sigma} < Z < \frac{b-\mu}{\sigma}\right)$$

Ex 8: Suppose *X* has a normal distribution with mean 30 and standard deviation 7. Find

a) $P(X < 35)$

b) $P(X > 18)$

c) $P(28 < X < 37)$

Ex 9: Pregnancy lengths have a normal distribution with a mean of 270 days and a standard deviation of 15 days. Find

a) the probability that a randomly selected pregnancy lasts longer than 261 days

b) the probability that a randomly selected pregnancy lasts between 267 days and 290 days

c) the probability that a randomly selected pregnancy lasts less than 242 days